

Educator's Guide

What Works in Teaching Math?

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Overview

This guide summarizes *Effective Programs in Elementary Mathematics: A Best-Evidence Synthesis* and *Effective Programs in Middle and High School Mathematics: A Best-Evidence Synthesis*, two research reviews conducted by Johns Hopkins University's Center for Research and Reform in Education.

The purpose of the reviews was to evaluate the achievement outcomes of alternative approaches to teaching math in elementary, middle, and high school. In both reviews, mathematics programs were grouped into the following three categories:

- Mathematics Curricula
- Computer-Assisted Instruction
- Instructional Process Programs

Study inclusion criteria for the two reviews included the use of randomized or well-matched control groups, study duration of at least 12 weeks, and the use of valid measures independent of treatments. A total of 189 studies met these criteria.

The key findings of the reviews were as follows:

- Programs designed to change daily teaching practices – particularly through the use of cooperative learning, classroom management, and motivation programs – have larger impacts on student achievement than programs that emphasize textbooks or technology alone.
- The most successful math programs encourage student interaction.

Background

The performance of U.S. students is neither disastrous nor stellar, and it is improving. In 2007, results from the Trends in International Mathematics and Science Study (TIMSS) showed that the average mathematics scores of both U.S. fourth-graders and eighth-graders were higher than the TIMSS scale average. In particular, the average U.S. fourth-grade mathematics score was higher than those scores of students in 23 of the 35 other countries included in the study, lower than those in eight countries (located in Asia or Europe), and not measurably different from those in the remaining four countries.

At eighth grade, the average U.S. mathematics score was higher than those scores of students in 37 of the 47 other countries included in the study, lower than those in five countries (located in Asia), and not measurably different than those in the other five countries.

Compared with 1995 TIMSS scores, the average mathematics scores for both U.S. fourth- and eighth-grade students were higher in 2007. At fourth grade, the U.S. average score in 2007 was 529, 11 points higher than its 1995 average. At eighth grade, the U.S. average mathematics score in 2007 was 508, 16 points higher than its 1995 average score.

However, while mathematics performance has grown substantially for all subgroups, the achievement gap between African American and Hispanic students and their White counterparts remains wide. In 2009, 51% of White fourth graders scored at or above “proficient” on the National Assessment of Education Progress (NAEP), but only 16% of African Americans and 22% of Hispanics scored this well.

One way to reduce mathematics achievement gaps, and to improve achievement overall, is to provide low-performing schools training and materials known to be markedly more effective than typical programs. Yet for such a strategy to be effective, it is essential to know what specific programs are most likely to improve mathematics achievement. In 2009, the U.S. Department of Education started rolling out an ambitious agenda for school reform, along with unprecedented resources behind the effort. This is an enormous opportunity for schools around the country, but educators need to have access to relevant and reliable information in order to make informed choices.

Goals of the Review

It has been difficult for educators to access clear and useful information about the evidence supporting alternative teaching strategies, and in particular it has been difficult to make comparisons between different programs of practice. However, as individual schools become increasingly responsible for choosing and implementing effective teaching methods, the evidence base for alternative approaches takes on even greater importance.

To help educators make informed decisions, this guide summarizes the evidence of effectiveness of alternative approaches to teaching mathematics, and uses a common set of procedures to evaluate and summarize the evidence. The following questions are addressed:

- What tools are available to improve the outcomes of students?
- Which textbooks, computer programs, and professional development techniques are most effective in the teaching and learning of math?

Review Methods

An exhaustive search for potentially relevant research considered hundreds of published and unpublished articles. Both reviews included studies from all countries, but the results had to be available in English. Studies also had to meet the following criteria:

- Schools or classrooms using each program had to be compared to randomly assigned or well-matched control groups.
- Measures had to be fair to all groups (and not, for example, be a test inherent to the program).
- Programs had to be evaluated for at least 12 weeks, and preferably a year or more.
- Outcome measures had to be assessments of the mathematics being taught in all classes.

Outcomes of individual studies are expressed as effect sizes, the proportion of a standard deviation by which the experimental group outperformed the control group (after adjusting for any pretest differences). Effect sizes of +0.20 or more are generally considered educationally significant (see glossary).

Key Findings

A total of 189 studies met the inclusion criteria across the two reviews.

Key findings were as follows:

- Programs designed to change daily teaching practices – particularly through the use of cooperative learning, classroom management, and motivations programs – have larger impacts on student achievement than programs that emphasize textbooks or technology alone.
- The most successful math programs encourage student interaction.

Findings by Category

In both reviews, mathematics programs were grouped into the following three categories: mathematics curricula, computer-assisted instruction, and instructional process programs.

Findings from each category were as follows:

1. Mathematics Curricula

A number of studies measured impact on achievement for a number of different curricula. These curricula fell into three categories:

- Innovative strategies that focus on problem-solving, alternative solutions, and conceptual understanding
- Traditional commercial textbooks
- A back-to-basics textbook that emphasizes a step-by-step approach



Our reviews assessed 13 studies of elementary mathematics curricula and 40 of middle and high school curricula. There was very little evidence that it mattered which curriculum was used, as none of the curricula showed any strong evidence of effectiveness. Although it might be suggested that the standardized tests used to measure performance would not detect some of the more sophisticated skills taught by some innovative curricula, there did not seem to be any evidence of this in the studies we looked at.

2. Computer-Assisted Instruction

In elementary schools, technology has typically been used as a supplement to classroom teaching, often used only a few times a week. Computer-assisted instruction (CAI) programs can help to identify children's strengths and weaknesses and then give them self-taught exercises designed to fill in any gaps.



Across the 38 studies of CAI programs that qualified for our reviews, we found that most studies found positive effects and none significantly favored a control group. There was not enough high quality evidence to recommend one program over another. We also found that the outcomes were stronger for computations than for concepts of problem solving. This finding is not surprising, as CAI programs particularly help children with their computation skills.

In middle and high schools, technology is used in three ways in the teaching of math:

- Supplemental programs, used to fill gaps in children’s knowledge
- Core programs, where the computer largely replaces the teacher
- Computer-managed learning systems, which use a computer to assess students and provide teachers with feedback for use in lessons

In the 40 qualifying studies that looked at these various programs, there was little evidence of effectiveness. No program stood out as having large and replicated effects, and computer-managed learning systems were particularly ineffective.

One limitation of both reviews is that many of the studies looked at programs that are no longer available. Technology is a rapidly developing area, and CAI programs are becoming ever more sophisticated. There is definitely a need for further research into the effectiveness of these programs.

3. Instructional Process Programs

A number of studies have looked at the impact of using extensive professional development to help teachers use effective teaching strategies. These studies keep the textbooks, content, and objectives the same, but change the teaching methods.

There were 36 qualifying studies of instructional process programs in elementary schools and 22 in middle and high schools. Professional development programs were found to have the strongest evidence of effectiveness. Cooperative learning was particularly strong. In cooperative learning, students work in pairs or small groups to help each other. Learning increases if the groups have a common goal that they can only achieve if all group members do well on independent learning. In other words, students have to teach each other, because their own success depends on it.



In elementary schools, programs that focused on classroom management and motivation also had strong evidence of effectiveness.

Conclusions

There are a number of important conclusions to be taken from this research. There is no evidence that different curricula give different outcomes in terms of achievement. Clearly this has important implications for the policy behind teaching math. There is also limited evidence that technology is effective, although more research on newer programs would help to improve our knowledge. However, there is strong evidence that using effective teaching strategies can make a real difference. Changing the way that children work together, and classroom management and motivation, can improve mathematics outcomes for all students.

It is clear that more high-quality studies are needed on the effectiveness of the curricula, textbooks, computer programs, and teaching strategies being used in schools today. The evidence reviewed was of variable quality and more work should be done to further investigate these findings. Nevertheless, there are a number of proven strategies that teachers can utilize to improve mathematics teaching and learning in their classes.

Also, it is important to note that the three types of approaches to mathematics instruction reviewed do not conflict with each other and may have additive effects if used together. However, findings suggest that educators, as well as researchers, might do well to focus more on how the classroom is organized to maximize student engagement and motivation, rather than expecting that choosing one or another textbook by itself will move students forward. In particular, both reviews found that the programs that produce consistently positive effects on achievement are those that fundamentally change what students do every day in their core math classes.

Implications

1. Teachers can significantly enhance mathematics learning by adopting cooperative learning.
2. Teachers can change their classroom management and motivation strategies to improve student outcomes.
3. Professional development works.
4. The evidence did not support the idea that different curricula give different outcomes in terms of mathematics achievement.
5. There is limited evidence in elementary schools – and even less evidence in middle and high schools – that CAI math programs are effective.

Program Ratings

Listed below are currently available programs, grouped by strength of evidence of effectiveness. For type, Mathematics Curricula = MC, Computer-Assisted Instruction = CAI, and Instructional Process Programs = IP. Ratings are provided for elementary math programs and middle and high school math programs.

Elementary Math Programs

Strong Evidence of Effectiveness

Rating	Program	Type	Description	Contact / Website
	Classwide Peer Tutoring	IP	Pair learning approach in which children take turns as teacher and learner.	Contact Charles Greenwood at greenwood@ku.edu
	Missouri Mathematics Program	IP	Program focusing on active teaching, classroom management, and motivation.	No contact information available.
	Peer Assisted Learning Strategies (PALS)	IP	Structured pair learning strategy in which children take turns as teachers and learners.	Website: www.kc.vanderbilt.edu/pals
	Student Teams-Achievement Divisions (now disseminated as PowerTeaching: Mathematics)	IP	Structured cooperative learning program in which students work in 4-member teams.	Website: www.successforall.org Contact: powerteaching@successforall.org
	TAI Math	IP/MC	Cooperative learning program in which students work on individualized materials in 4-member teams.	Contact Brent Farmer, Charlesbridge Publishing, 800-225-3214, or bfarmer@charlesbridge.com

Moderate Evidence of Effectiveness

Rating	Program	Type	Description	Contact / Website
	Classworks	CAI	Supplementary integrated learning system.	Website: www.curriculumadvantage.com
	Cognitively Guided Instruction	IP	Program that provides teachers with workshops in math strategies.	Contact Linda Levi, Teachers Development Group, at lindalevi@teachersdg.org
	Connecting Math Concepts	IP/MC	Structured approach to math with grouping by performance.	Website: www.sraonline.com/math
	Consistency Management & Cooperative Discipline	IP	Program that emphasizes classroom management, student engagement.	Contact Jerome Freiberg, University of Houston, at cmcd@uh.edu .
	Project SEED	IP	Supplementary program that has mathematicians teach advanced topics in math to supplement regular instruction.	Website: www.projectseed.org
	Small-Group Tutoring	IP	Provides tutoring in small groups for struggling first graders.	Contact Lynn Fuchs, Vanderbilt University, at lynn.fuchs@vanderbilt.edu

Limited Evidence of Effectiveness

Rating	Program	Type	Description	Contact / Website
	Accelerated Mathematics	CAI	Supplementary program that prints out assignments for students based on their level of performance.	Website: www.renlearn.com/mathrenaissance
	Dynamic Pedagogy	IP	Program that provides teachers with workshops in math strategies.	Contact Eleanor Armour-Thomas at armourthomas@yahoo.com
	Every Day Counts	IP	An interactive K-6 bulletin-board program designed to supplement ordinary math instruction with discussions about math concepts built around the calendar and other classroom routines.	Website: www.greatsource.com
	Excel Math	MC	K-6 math curriculum that focuses on problem solving, integrated lessons, and development of thinking skills.	Website: www.excelmath.com
	Everyday Mathematics	MC	NSF-supported curriculum that emphasizes problem solving and concepts.	Website: www.wrightgroup.com or http://everydaymath.uchicago.edu/
	Growing with Mathematics	MC	Core mathematics program for PreK-5.	Website: www.wrightgroup.com

Rating	Program	Type	Description	Contact / Website
	Houghton-Mifflin Mathematics	MC	Standard math curriculum that has a focus on skill building, problem solving, and concept mastery.	Website: www.eduplace.com/math
	Knowing Mathematics	MC	Remedial program for students performing below grade level.	Website: http://www.eduplace.com/profdev/knowning1
	Mastery Learning	IP	A strategy in which time to learn is adjusted to fit aptitude. Students proceed to new material only after basic prerequisite material is mastered.	No contact information available.
	Lightspan	CAI	Supplementary integrated learning system. Also provides CAI programs for home use	Website: www.plato.com (Note: Lightspan and Plato Learning have merged)
	Project CHILD	IP/CAI	Program that uses cooperative learning, multi-age grouping, extensive computer-assisted instruction, and other features.	Website: www.ifs.org/projectchild/

Insufficient Evidence of Effectiveness

Math Steps
 Math Trailblazers
 Saxon Math
 Scott Foresman-Addison Wesley Mathematics

N No Qualifying Studies

Adventures of Jasper Woodbury
AIMSweb® Pro Math
Bridges in Mathematics
Compass Learning (current version)
Corrective Math
Count, Notice, & Remember
Destination Math Series
First in Math®
Great Explorations in Math and Science
Harcourt Math
Investigations in Number, Data, and Space
Larson's Elementary Math
Math Advantage
MathAmigo
Math Blasters
Math Central
Math Coach
Math Expressions
Math Explorations and Applications
Math in My World
Math Made Easy
Math Matters
Math Their Way
Math & Me Series
Math & Music
Mathematics Plus
Mathematics Their Way
Mathletics
Math Realm
MathWings
Macmillan McGraw-Hill Math
McGraw-Hill Mathematics
Number Power
Problem Solving Step by Step
Progress in Mathematics
Project IMPACT
Project M3: Mentoring Mathematical Minds
Rational Number Project
Real Math

Reciprocal Peer Tutoring
 Scott Foresman Math Around the Clock
 Singapore Math
 Skills Tutor/Cornerstone2
 SuccessMaker (Current version)
 TIPS Math
 Voyages
 Waterford Early Math
 Yearly Progress Pro

Middle and High School Math Programs

Strong Evidence of Effectiveness

Rating	Program	Type	Description	Contact / Website
	IMPROVE	IP-Cooperative Learning	A program that combines cooperative learning, metacognitive instruction, and mastery learning that accommodates student diversity in a heterogeneous classroom.	E-mail: mevarz@mail.biu.ac.il
	Student Teams-Achievement Divisions (STAD, now disseminated as PowerTeaching: Mathematics)	IP-Cooperative Learning	A cooperative learning program in which students work in 4-member heterogeneous groups to help each other master academic content. Teachers follow a schedule of teaching, team work, and individual assessment.	Website: www.successforall.org Contact: powerteaching@successforall.org

Moderate Evidence of Effectiveness

 None

Limited Evidence of Effectiveness

Rating	Program	Type	Description	Contact / Website
	Cognitive Tutor	CAI	An intelligent tutoring system that emphasizes algebra problem solving. Working on computers, students carry out investigations of real-world problems using spreadsheets, graphers, and symbolic calculators.	E-mail: help@carnegielearning.com Website: www.carnegielearning.com
	Core-Plus Mathematics	MC	Integrated mathematics curriculum that emphasizes applications and mathematical modeling, use of graphing calculators, and small-group collaborative learning through problem-based investigations.	E-mail: cpmp@wmich.edu Website: www.wmich.edu/cpmp
	Expert Mathematician	CAI	A program in which students are taught to use the LOGO programming language and proceed through a constructivist, integrated series of computer and workbook activities emphasizing problem solving and creativity.	Complete contact form at: www.expertmath.org/contact.html Website: www.expertmath.org

Rating	Program	Type	Description	Contact / Website
	Jostens	CAI	Provides an extensive set of assessments which place students according to their current levels of performance and then gives students exercises designed primarily to fill in gaps in their skills.	Complete contact form at: www.compasslearning.com/Contact/Default.aspx Website: www.compasslearning.com
	Math Thematics	MC	Encourages students to investigate mathematical concepts through exploratory, activity based learning.	Complete contact form at: www.classzone.com/cz/contact_us.htm Website: www.classzone.com/books/math_thematics1/
	Partnership for Access to Higher Mathematics (PATH)	IP	A program for at-risk eighth graders that focuses on improving curriculum and instruction with use of constructivist approaches, manipulatives, and technology.	No website available.
	Plato	CAI	An integrated learning system that has been evaluated as a remedial program.	Complete contact form at: www.plato.com/Contact-Us/Forms/K-12-Learning-Request-For-Information.aspx Website: www.plato.com
	Prentice-Hall Course 2	MC	A traditional, seventh grade curriculum that emphasizes proportional reasoning.	Complete contact form at: www.k12pearson.com/contactus/contact_default.cfm?cmpy=PH Website: www.phschool.com/home.html

Rating	Program	Type	Description	Contact / Website
	Saxon Math	MC	A program that emphasizes teaching in small, incremental steps, ensuring mastery of each concept before the next step is introduced.	E-mail: info@SaxonPublishers.com Website: saxonpublishers.harcourtachieve.com
	Talent Development Mathematics	IP	Standards-based curriculum combined with computer-based mathematics that develops advanced skills in geometry, data, and algebra.	E-mail: lmuskauski@csos.jhu.edu Website: www.csos.jhu.edu/tdhs

Insufficient Evidence of Effectiveness

Accelerated Math
 Connected Mathematics
 I Can Learn
 Interactive Mathematics Program
 Learning Logic Lab
 Mastery Learning
 Mathematics in Context
 McDougal-Littell
 PALS/CBM
 Prentice Hall Algebra
 SIMMS Integrated Mathematics
 University of Chicago School Mathematics Project (UCSMP)

N No Qualifying Studies

Adventures of Jasper Woodbury Series
 AquaMOOSE
 CAP Mnemonic Instruction
 College Preparatory Mathematics
 Compass Learning
 Connecting Math Concepts
 Concepts in Algebra, Everyday Learning

CORD Contextual Mathematics, CORD Applied Mathematics, CORD Algebra 1
Corrective Mathematics
Destination Math
Focus on Algebra, Addison Wesley Longman
Fun Math
Generalizable Mathematics Skills Instructional Intervention
Geometric Supposers
Glencoe Mathematics & Pre-Algebra
Hawaii Algebra Learning Project (HALP)
Heath Mathematics Connection
Heath Passport to Mathematics
Introducing Math Teachers to Inquiry
Mastering Fractions
Math Advantage
Math and Science Academy
Math Blaster Mystery
MATH Connections
Math Corps Summer Camp
Math Matters
Mathematics in Context (6-8)
Mathematics: Modeling our World, COMAP/ARISE
Mathematics Plus
MathFacts
MathScape
MathStar
McGraw-Hill Algebra 1
Middle Grade Mathematics Renaissance
Middle School Family Math
Middle School Math through Applications
Model Mathematics Program
Moving With Math
Multimedia Probability & Statistics
Orchard Software
Pacesetter
Peoria Urban Mathematics Plan for Algebra
Powerful Connections
Project AutoMath
QUASAR Project
Rice University School Mathematics Project
Saturday Academy
Scott Foresman Middle School Math

SmartHelp
Southern California Regional Algebra Project
SuccessMaker, CCC
TASS Tutorial Program, Blitz
TGT (Teams-Games-Tournament)
Transition to Geometry (summer program)
University of Illinois at Chicago All Learn Mathematics
Voyager Math
Wayang Outpost Interactive Tutoring System
Word Problem Solving Tutor, Apangea

Full Report

This guide is adapted from the following full reports:

- Slavin, R.E., & Lake, C. (2008). Effective programs for elementary mathematics: A best evidence synthesis. *Review of Educational Research*, 78 (3), 427-515.
- Slavin, R.E., Lake, C., & Groff, C. (2009). Effective programs in middle and high school mathematics: A best evidence synthesis. *Review of Educational Research*, 79 (2), 839 – 911.

To access the full reports, visit Johns Hopkins University’s Best Evidence Encyclopedia (BEE) website at www.bestevidence.org.

Glossary

Effect size

The effect size shows how much difference a program/intervention makes. It is the difference between the mean of the experimental group and the mean of the control group, divided by the standard deviation of the control group.

The important point is that the larger the effect size, the greater the difference the program/intervention has made. An effect size of more than +0.20 is generally considered educationally significant.

References

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Katsberg, D., & Brenwald, S. (2008) *Highlights From TIMSS 2007: Mathematics and Science Achievement of U.S. Fourth- and Eighth-Grade Students in an International Context* (NCES 2009001). U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.

National Assessment of Educational Progress (2009). *The nation's report card*. Washington, DC: National Center for Education Statistics.

Slavin, R.E., & Lake, C. (2008). Effective programs for elementary mathematics: A best evidence synthesis. *Review of Educational Research*, 78 (3), 427-515.

Slavin, R.E., Lake, C., & Groff, C. (2009). Effective programs in middle and high school mathematics: A best evidence synthesis. *Review of Educational Research*, 79 (2), 839 – 911.